**Mathematics: Number Operations Learning Progression**

| **Domain: Mathematics** | | | | | |
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| **Strand: Operations and Algebraic Thinking** | | | | | |
| **Learning Progression: Number Operations** | | | | | |
| **Operational Definition: Uses objects to understand addition as putting together and adding to, and to understand subtraction as taking apart and taking from** | | | | | |
|  | **Level 1** | **Level 2** | **Level 3** | **Level 4** | **Level 5** |

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| **Addition Problems** |  | Adds one more to a small set of objects (totals up to 4) to make a set that is *more*. | Solves simple addition problems (totals up to 5) by joining two small sets of objects and counting the total. | Solves simple addition problems (totals up to 10) by joining two sets of like objects (e.g., apples and apples) or two sets of related objects (e.g., apples and oranges) and counting the total. | Solves simple addition problems (totals up to 10), using objects, fingers, or drawings, starting from a given number and counting on to find the sum. |
| **Subtraction Problems** |  | Takes one away from a small set of objects (up to 5) to make a set that is *less.* | Solves simple subtraction problems by taking a given amount from a set of objects (up to 5) and counting the remaining objects. | Solves simple subtraction problems (totals up to 10) by matching objects from two sets of related objects (e.g., cups and saucers) and counting the objects that do not have a match. | Solves simple subtraction problems (totals up to 10), using objects, fingers, or drawings, and appropriate strategies (e.g., counting back). |
| **Decomposing Numbers** |  |  |  | Decomposes numbers (up to 5) into two groups in more than one way, using objects or drawings, and records each with a drawing. | Decomposes numbers (up to 10) into two groups in more than one way, using objects or drawings, and records each with a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). |
| **Completing a Set** |  |  |  | Finds and identifies the amount needed to complete a set (totals up to 5), using objects. | Finds and identifies the amount needed to complete a set (totals up to 10), using objects, fingers, or drawings. |

**Addition Problems**

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| **Directions** | **Rubric** | | **Evidence Examples** |
| Observe children during the regular daily routine and look for instances when they recognize differences in quantity between sets of objects and solve addition problems.  For each child, pay attention to the following details as you observe:   * what objects comprise the sets * what quantities comprise each set * what the total is when the sets are combined * whether the child relies on concrete objects or uses some other strategy to solve the addition problem(s) (e.g., creating representational drawings) * whether the child counts on from the amount in one set to find the total   Collect and record evidence for each instance and each child that you observe. Use the evidence to determine and record each child’s level. | **2** | Child adds one more to a small set of objects (totals up to 4) to make a set that is *more*. | **Carlos—**Carlos decided to share his special markers with his friend Lee. He gave Lee two markers and Lee said, “Can I have more, Carlos?” Carlos handed Lee one more marker and said, “Now you have three.” |
| **3** | Child solves simple addition problems (totals up to 5) by joining two small sets of objects and counting the total. | **Joan—**Joan and Lulu were playing with trucks on separate ends of the carpet; Joan drove her two trucks to meet Lulu’s three trucks. Joan counted all the trucks and said, “Now we have five trucks!” |
| **4** | Child solves simple addition problems (totals up to 10) by joining two sets of like objects (e.g., apples and apples) or two sets of related objects (e.g., apples and oranges) and counting the total. | **Lindsey—**Lindsey was in charge of distributing pencils at the blue and red tables and said, “There are four kids at the blue table and four kids at the red table.” She stopped to count all the kids, touching each one gently as she counted. She said, “There are eight kids so I need eight pencils.” |
| **5** | Child solves simple addition problems (totals up to 10), using objects, fingers, or drawings, starting from a given number and counting on to find the sum. | **Jim—**Jim was working to solve the addition problem 4 + 5. He counted out four fingers on one hand and all five on the other hand. He looked at the hand with all five fingers up and said, “Five.” Then he wiggled and counted each finger that was up on the other hand and said, “Six, seven, eight, nine. Four plus five equals nine.” |

**Subtraction Problems**

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| **Directions** | **Rubric** | | **Evidence Examples** |
| Observe children during the regular daily routine and look for instances when they solve subtraction problems.  For each child, pay attention to the following details as you observe:   * what objects comprise the set(s) the child may be using to solve the problem(s) * what quantities comprise each set (if applicable) * what quantity the child is taking away * whether or not the child relies on concrete objects or uses some other strategy to solve the subtraction problem(s) (e.g., creating representational drawings or counting on)   Collect and record evidence for each instance and each child that you observe. Use the evidence to determine and record each child’s level. | **2** | Child takes one away from a small set of objects (up to 5) to make a set that is *less.* | **Dylan—**Dylan and Barbara were building a tower with blocks. He stacked four rectangular blocks end to end and the tower began to wobble on the uneven carpet surface. Barbara said, “That’s too many.” Dylan took off one block and said, “Now there’s less.” |
| **3** | Child solves simple subtraction problems by taking a given amount from a set of objects (up to 5) and counting the remaining objects. | **Paula—**Paula was eating crackers for her lunch. She counted five crackers at the start of lunch. Then, she ate two crackers. Next, she counted to see how many crackers she had left. She said, “I had five and now I have three.” |
| **4** | Child solves simple subtraction problems (totals up to 10) by matching objects from two sets of related objects (e.g., cups and saucers) and counting the objects that do not have a match. | **Gilbert—**Gilbert was given some straws to put on the table with the ten milk cartons that were there. He placed one straw by each of the first six cartons. Then he counted the remaining cartons and said, “Mrs. Garcia, I need four more straws.” |
| **5** | Child solves simple subtraction problems (totals up to 10), using objects, fingers, or drawings, and appropriate strategies (e.g., counting back). | **Jessenia—**Jessenia was working to solve the subtraction problem 10 – 7. She held out all ten fingers and counted down from seven until she had three fingers left. Then she said, “Ten minus seven equals three.” |

**Decomposing Numbers**

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| **Directions** | **Rubric** | | **Evidence Examples** |
| Observe children during the regular daily routine and look for instances when they decompose numbers.  For each child, pay attention to the following details as you observe:   * what number the child is decomposing * how many groups the child forms during decomposition * how many ways the child decomposes the number   Collect and record evidence for each instance and each child that you observe. Use the evidence to determine and record each child’s level. | **4** | Child decomposes numbers (up to 5) into two groups in more than one way, using objects or drawings, and records each with a drawing. | **Franklin—**Franklin wanted to share his four cars with his friend Arnie. He put three cars in one group and one car in another group and said, “I could give Arnie three and keep one.” He frowned and put the four cars back together in one group. Then he put two cars in one group and two cars in the other group and said, “Now Arnie will have two cars and I will have two cars too.” |
| **5** | Child decomposes numbers (up to 10) into two groups in more than one way, using objects or drawings, and records each with a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). | **Nuray—**Nuray and Cynthia were pretending to run a restaurant. Emeril, who was pretending to be a customer, wanted to know how much food he could get for ten dollars. Cynthia said, “Well, all we have are pickles and hot dogs. It’s one dollar for a pickle and one dollar for a hot dog.” Then Nuray said, “You could get five pickles and five hot dogs OR you could get nine pickles and one hot dog. You could even have six pickles and four hot dogs.” She wrote down the options for him on her pretend waitress notebook:  5P + 5H = 10  9P + 1H = 10  6P + 4H = 10 |

**Completing a Set**

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| **Directions** | **Rubric** | | **Evidence Examples** |
| Observe children during the regular daily routine and look for instances when they complete a set.  For each child, pay attention to the following details as you observe:   * what quantity of objects make up a complete set * what the quantity of objects are in the starting set * what quantity of objects the child needs to add to complete the set * how the child works to complete the set   Collect and record evidence for each instance and each child that you observe. Use the evidence to determine and record each child’s level. | **4** | Child finds and identifies the amount needed to complete a set (totals up to 5), using objects. | **Tina—**Tina was making a picture with squares and circles. She told Lupita, “I have three circles but I need five to make my picture. Can I have two of your circles if you don’t use them?” |
| **5** | Child finds and identifies the amount needed to complete a set (totals up to 10), using objects, fingers, or drawings. | **Zander—**Zander was carrying two balls out to the field when the gym teacher said, “Zander, we need eight balls to play the game today.” Zander looked at the two balls in his arms, placed them carefully on the ground, and counted on from two on his fingers until he got to eight. He said, “Three, four, five, six, seven, eight” and put a finger out for each number he said. When he got to eight, he looked at his fingers and said, “I’ll go get six more.” |